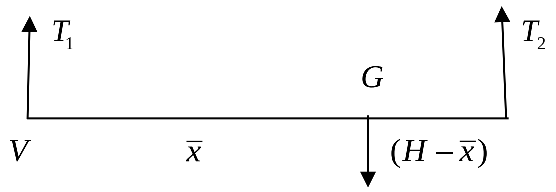
Mark Scheme (Results)

Summer 2023

Pearson Edexcel International Advanced Level
In Mechanics M3 (WM03)
Paper 01

Question Number	Scheme	Marks
1.	$\int_0^3 \sqrt{(x+1)}\,dx$	M1
	$= \frac{2}{3}\left[(x+1)^{\frac{3}{2}}\right]_0^3$	A1
	$\dfrac{\int_0^3 \frac{1}{2}\left(\sqrt{(x+1)}\right)^2 dx}{\int_0^3 \sqrt{(x+1)}\,dx}$ or $\dfrac{\int_0^3 \frac{1}{2}(x+1)\,dx}{\int_0^3 \sqrt{(x+1)}\,dx}$	M1
	$= \dfrac{\frac{1}{2}\left[\frac{1}{2}x^2 + x\right]_0^3}{\frac{2}{3}\left[(x+1)^{\frac{3}{2}}\right]_0^3}$ or $\dfrac{\frac{1}{2}\left[\frac{1}{2}(x+1)^2\right]_0^3}{\frac{2}{3}\left[(x+1)^{\frac{3}{2}}\right]_0^3}$	A1
	$= \dfrac{45}{56}$ (0.80 or better)	A1 (5)
		(5)
	Notes	
M1	Use of $\int_0^3 \sqrt{(x+1)}\,dx$. Limits not needed. Accept $k \times \int_0^3 \sqrt{(x+1)}\,dx$ where k is a constant. To give the mark for 'use' we must see an attempt at integration. An attempt at integration is seen when the powers increase by 1.	
A1	Correct integrated expression with correct limits	
M1	Use of $\dfrac{\int_0^3 \frac{1}{2}\left(\sqrt{(x+1)}\right)^2 dx}{\int_0^3 \sqrt{(x+1)}\,dx}$. Limits not needed. The formula must be correct but allow a constant multiple if it appears on both numerator and denominator. We must see the correct formula and an attempt at integrating the numerator	
A1	Correct integrated expression for the numerator in the correct formula with correct limits.	
A1	Correct answer. This question comes with a calculator warning: the correct answer must come from integrated expressions ie both previous A's must have been awarded. Numerical substitution does not need to be seen.	

Question Number	Scheme	Marks
2.	$T = mg\cos\theta$	M1A1
	$T = \dfrac{2mg\left(\frac{21}{10}a - ka\right)}{ka}$	M1A1
	$\frac{4}{5}mg = \dfrac{2mg\left(\frac{21}{10}a - ka\right)}{ka}$	dM1
	$k = \frac{3}{2}$ or 1.5	A1
		(6)
	Notes	
M1	Resolve parallel to the string, correct no. of terms, condone sign errors and sin/cos confusion (**or** resolve in two directions and eliminate the unknown force **or** use trig on a right-angled triangle of forces) to give an equation in T, mg and θ **only**	
A1	Correct equation. Trig does not need to be substituted.	
M1	Use Hooke's Law with correct structure.	
A1	Correct equation	
dM1	Substitute trig and eliminate T to produce equation in k only, dependent on previous M's. If x is used for extension, should see $x = \frac{2k}{5}a \rightarrow ka + \frac{2k}{5}a = \frac{21}{10}a$	
A1	cao	
ALT 1	**First M1A1**	
M1A1	Complete method to form an equation in T and θ Vert: $mg = T\cos\theta + F\sin\theta$ Horiz: $F\cos\theta = T\sin\theta$ Eliminate F eg $\dfrac{\sin\theta}{\cos\theta} = \dfrac{mg - T\cos\theta}{T\sin\theta}$	

Question Number	Scheme	Marks
3(a)	Mass: $\frac{1}{3}\pi r^2 H$ $\quad$ $\frac{1}{3}\pi r^2 h$ $\quad$ $\frac{1}{3}\pi r^2 H - \frac{1}{3}\pi r^2 h$	B1
	Distance from V: $\frac{3H}{4}$ $\quad$ $H - \frac{1}{4}h$ $\quad$ $\bar{x}$	B1
	$\frac{1}{3}\pi r^2 H \times \frac{3H}{4} - \frac{1}{3}\pi r^2 h \times \left(H - \frac{1}{4}h\right) = \left(\frac{1}{3}\pi r^2 H - \frac{1}{3}\pi r^2 h\right)\bar{x}$	M1A1
	$\bar{x} = \frac{(3H-h)(H-h)}{4(H-h)} = \frac{1}{4}(3H-h)$ *	A1*
		(5)
3(b)		
	M(G), $T_1\bar{x} = T_2(H-\bar{x})$	M1
	$\frac{T_1}{T_2} = \frac{H - \frac{1}{4}(3H-h)}{\frac{1}{4}(3H-h)}$	A1
	$\frac{T_1}{T_2} = \frac{H+h}{3H-h}$	A1 (3)
		(8)
	Notes	
3(a)		
B1	Three correct mass ratios: $H \quad h \quad H - h$	
B1	Three correct distances (Allow if measured from some other axis)	
M1	Moments equation with correct no. of terms, dim correct. Condone addition, treat as a sign error. Must be working with solids eg not a conical shell.	
A1	Correct unsimplified equation (For their axis)	
A1*	Given answer correctly obtained, including cancelling ($H – h$). A factorised expression should be seen to reach the GIVEN answer.	
3(b)		
M1	Complete method to obtain an equation in H, h, T_1 and T_2 only . Must be dimensionally correct. May take moments about G and substitute for $\bar{x}$. Alternatively may use two equations, eliminate weight and substitute for $\bar{x}$.	
A1	Correct equation in T_1, T_2, H and h only	
A1	Correct answer. The question asks for simplest form.	
3(a) ALT 1	**Distances measured from circular face**	
B1	Mass: $\frac{1}{3}\pi r^2 H$ $\quad$ $\frac{1}{3}\pi r^2 h$ $\quad$ $\frac{1}{3}\pi r^2 H - \frac{1}{3}\pi r^2 h$	
B1	Dist from Circular face: $\frac{H}{4}$ $\quad$ $\frac{1}{4}h$ $\quad$ d	
M1A1	$\frac{1}{3}\pi r^2 H \times \frac{H}{4} - \frac{1}{3}\pi r^2 h \times \frac{1}{4}h = \left(\frac{1}{3}\pi r^2 H - \frac{1}{3}\pi r^2 h\right)d$	

Question Number	Scheme	Marks
A1*	Leads to $d=\dfrac{H+h}{4}$ which must be subtracted from H to reach the required distance: $\bar{x}=H-\dfrac{H+h}{4}=\dfrac{1}{4}(3H-h)$ *	
3(b) ALT 1		
M1	May use two equations and eliminate weight/mass ratio to find an equation in terms of T_1, T_2, H and h only. Vert $T_1+T_2=W$ M(V), $W\bar{x}=T_2H$ M(circular face), $T_1H=W(H-\bar{x})$ Eliminate W	

Question Number	Scheme	Marks
4(a)	$R\cos\alpha = mg$	M1 A1
	$R\sin\alpha = \dfrac{m\left(\frac{1}{4}gr\right)}{r}$ **OR:** $mg\sin\alpha = \dfrac{m\left(\frac{1}{4}gr\right)}{r}\cos\alpha$ M2 A2	M1A1
	$\tan\alpha = \dfrac{1}{4}$ *	A1*
		(5)
4(b)	Vert equil: $S\cos\alpha - F\sin\alpha = mg$ Perp N2L: $S - mg\cos\alpha = \dfrac{mV^2}{r}\sin\alpha$	M1A1
	N2L towards O: $S\sin\alpha + F\cos\alpha = \dfrac{mV^2}{r}$ Parallel N2L: $F + mg\sin\alpha = \dfrac{mV^2}{r}\cos\alpha$	M1A1
	$F = \mu S$	B1
	Eliminate F, sub for trig and solve for V in terms of μ, r and g.	dM1
	$V = \sqrt{rg\dfrac{(1+4\mu)}{(4-\mu)}}$ oe	A1
		(7)
		(12)
	Notes	
4(a)		
	Note: For use of θ instead of α in (a) penalise only the last mark in (a). The maximum score is M1A1M1A1A0*	
M1	Resolve vertically correct no. of terms, condone sign errors and sin/cos confusion.	
A1	Correct equation	
M1	Equation of motion horizontally correct no. of terms, condone sign errors and sin/cos confusion. V does not need to be substituted. Allow $r\omega^2$ for acceleration but not 'a'.	
A1	Correct equation. May still contain V and either form of acceleration (circular).	
OR	M2 Equation of motion down the plane correct no. of terms, condone sign errors and sin/cos confusion. A1 Correct equation with at most one error A1 Correct equation	
A1*	Correctly obtain given answer, written exactly.	
4(b)		
M1	Resolve vertically or equation of motions perpendicular. Correct no. of terms, condone sign errors and sin/cos confusion. M0 if R from (a) is used.	

Question Number	Scheme	Marks
A1	Correct equation	
M1	Equation of motion horizontally or parallel to slope. Correct no. of terms, condone sign errors and sin/cos confusion. M0 if *R* from (a) is used.	
A1	Correct equation	
B1	$F = \mu S$ seen where *S* is the normal reaction in (b).	
dM1	Eliminate *F*, sub for trig and solve for *V* in terms of μ, *r* and *g*. Dependent on both previous M marks.	
A1	Correct answer.	

Question number	Scheme	Marks
5(a)	$v\dfrac{dv}{dx}=-\dfrac{gR^2}{x^2}$	M1 A1
	$\int v\ dv=-\int\dfrac{gR^2}{x^2}\ dx$ or $\dfrac{1}{2}V^2=-\int\dfrac{gR^2}{x^2}\ dx$ Or the Energy alternative below	M1
	$\dfrac{1}{2}v^2=\dfrac{gR^2}{x}+C$	A1
	Use of $x = R$, $v = U$ to find C $\quad (C=\dfrac{1}{2}u^2-gR)$	M1
	$v^2=\dfrac{2gR^2}{x}+U^2-2gR$ *	A1*
		(6)
5(b)	$\dfrac{1}{4}gR=\dfrac{2gR^2}{x}+gR-2gR$	M1
	$x=\dfrac{8R}{5}$	A1
	$AB=\dfrac{3R}{5}$ oe	A1
		(3)
5(c)	Correct statement regarding $\dfrac{2gR^2}{x}$ for example • $\dfrac{2gR^2}{x}>0$ for $x\ge R$ • $x\to\infty,\ \dfrac{2gR^2}{x}\to 0$	M1
	Correct reasoning. • $U^2-2gR=0$ • $U^2\to 2gR$ • $U^2\ge 2gR$	dM1
	$U_{\text{MIN}}=\sqrt{2gR}$	A1
		(3)
		(12)

	Notes
5(a)	
M1	Equation with or without -ve sign and any derivative form for the acceleration
A1	Correct equation with -ve sign
M1	Separate variables and clear attempt to integrate acceleration in terms of v and x.
A1	Correct equation; allow without C
M1	Use of initial conditions or limits
A1*	Given answer correctly obtained

5(b)	
M1	Substitution of v^2 and U^2 into (a) to produce a correct equation
A1	Correct value of x
A1	cao
5(c)	
M1	Correct reasoning for the term $\frac{2gR^2}{x}$ Accept $x = \infty$, $x \to \infty$, $\frac{2gR^2}{x} = 0$
dM1	Dependent on previous M. Correct reasoning leading to correct equation or inequality.
A1	cso
ALT 5a	**Energy approach must use integration**
M1 A1	Energy equation with variable force. The sign may be missing for the M mark. $\frac{1}{2}mv^2 - \frac{1}{2}mU^2 = \int F \text{ dx} = \int -\frac{mgR^2}{x^2} \text{ dx}$
M1 A1	Clear attempt to integrate. Limits may be missing or incorrect. $\frac{1}{2}mv^2 - \frac{1}{2}mU^2 = \left[\frac{mgR^2}{x}\right]_R^x$
M1	Correct limits substituted the right way round. $\frac{1}{2}mv^2 - \frac{1}{2}mU^2 = \frac{mgR^2}{x} - \frac{mgR^2}{R}$
	$v^2 - U^2 = \frac{2gR^2}{x} - 2gR$
A1*	$v^2 = \frac{2gR^2}{x} + U^2 - 2gR$

Question number	Scheme	Marks
6(a)	$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mga\sin\theta$	M1A1A1
	$T - mg\sin\theta = \frac{mv^2}{a}$	M1A1A1
	$T = \frac{mu^2}{a} + 3mg\sin\theta$ *	A1*
		(7)
(b)	$0 = \frac{m(\frac{12ag}{5})}{a} + 3mg\sin\theta$	M1
	$\sin\theta = -\frac{4}{5}$	A1
	$\frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{12ag}{5}\right) = mga \times -\frac{4}{5}$ **OR** $0 - mg \times -\frac{4}{5} = \frac{mv^2}{a}$	M1
	$v = 2\sqrt{\frac{ag}{5}}$, $0.89\sqrt{ag}$ or better	A1
		(4)
(c)	Vertical motion $0 = \left(2\sqrt{\frac{ag}{5}} \times \frac{3}{5}\right)^2 - 2gh$ OR Energy $mgh = \frac{1}{2}m\left(2\sqrt{\frac{ag}{5}}\right)^2 - \frac{1}{2}m\left(2\sqrt{\frac{ag}{5}} \times \frac{4}{5}\right)^2$	M1A1ft
	$h = \frac{18a}{125}$	A1
	H, height above O $= h - a\sin\theta = \frac{18a}{125} + \frac{4a}{5}$	dM1
	$= \frac{118a}{125}$, $0.94a$, $0.944a$	A1
		(5)
	OR using energy from start to top	
	$mgH = \frac{1}{2}m\left\{\frac{12ag}{5} - \left(2\sqrt{\frac{ag}{5}} \times \frac{4}{5}\right)^2\right\}$	M2A1ft A1
	$H = \frac{118a}{125}$, $0.94a$, $0.944a$	A1
		(5)
		(16)
	Notes	
6(a)		
M1	Energy equation with correct no. of terms, dim correct. May use h instead of $a\sin\theta$	
A1	Correct equation with at most one error	

A1	Correct equation
M1	Equation of motion towards *O*, correct no. of terms, condone sign errors and sin/cos confusion. Accept acceleration in either circular form but do not accept 'a'. The radius may be given as *r*.
A1	Equation with at most one error
A1	Correct equation
A1*	Given answer correctly obtained and written exactly as printed.
6(b)	
M1	Put $T = 0$ and $u = 2\sqrt{\frac{3ag}{5}}$
A1	Correct value of $\sin\theta$
M1	Put $u = 2\sqrt{\frac{3ag}{5}}$ and their $\sin\theta$ into energy equation **OR** put $T = 0$ and their $\sin\theta$ into equation of motion
A1	Correct answer
6(c)	If an energy approach is used in (c) the equation must have 2 KE terms, one of which must have a sin/cos component included.
M1	Use vertical motion or energy to obtain an equation in *h* only. A component of speed must be used for either approach.
A1ft	Correct equation ft on their answer to (b).
A1	Correct value of *h*
dM1	Correct method to find *H*. Dependent on previous M.
A1	cao
	OR
M2	Complete method to obtain an equation in *H* only (must be using horizontal cpt of velocity at the top)
A1ft	Correct equation with at most one error
A1	Correct equation
A1	Correct answer

Question number	Scheme	Marks
7(a)	$\dfrac{\lambda(D-l)}{l} = mg$	M1A1
	$\dfrac{\lambda(2l)^2}{2l} = mg \times 3l$	M1A1A1
	$D = \dfrac{5l}{3}$ *	A1*
		(6)
7(b)	$mg - T = m\ddot{x}$ or $T - mg = m\ddot{x}$	M1
	$mg - \dfrac{3mg}{2l}(\dfrac{2l}{3} + x) = m\ddot{x}$ or $\dfrac{3mg}{2l}(\dfrac{2l}{3} - x) - mg = m\ddot{x}$	dM1A1
	$-\dfrac{3g}{2l}x = \ddot{x}$ hence SHM	A1
	period = $\dfrac{2\pi}{\omega} = \dfrac{2\pi}{\sqrt{\dfrac{3g}{2l}}}$ $\{\ \omega = \sqrt{\dfrac{3g}{2l}}\ \}$	M1
	$= 2\pi\sqrt{\dfrac{2l}{3g}}$ *	A1*
		(6)
7(c)	$-\dfrac{2l}{3} = \dfrac{4l}{3}\cos\sqrt{\dfrac{3g}{2l}}t$	M1A1A1
	$t = \dfrac{2\pi}{3}\sqrt{\dfrac{2l}{3g}}$	A1
	OR	
	Complete method $t = \dfrac{1}{4}2\pi\sqrt{\dfrac{2l}{3g}} + t_1$ where $\dfrac{2l}{3} = \dfrac{4l}{3}\sin\sqrt{\dfrac{3g}{2l}}t_1$	M1A1A1
	$t = \dfrac{2\pi}{3}\sqrt{\dfrac{2l}{3g}}$ oe	A1
	OR	
	Complete method $t = \dfrac{1}{2}2\pi\sqrt{\dfrac{2l}{3g}} - t_1$ where $\dfrac{2l}{3} = \dfrac{4l}{3}\cos\sqrt{\dfrac{3g}{2l}}t_1$	M1A1A1
	$t = \dfrac{2\pi}{3}\sqrt{\dfrac{2l}{3g}}$ or equivalent exact form.	A1
		(4)
		(16)
	Notes	

7(a)	
M1	Use Hooke's law in D and equate to mg
A1	Correct equation

M1	Energy equation with correct no. of terms. EPE of the form $\frac{\lambda x^2}{kl}, k \neq 1$
A1	Equation with at most one error
A1	Correct equation
A1*	Given answer correctly obtained
7(b)	
M1	Equation of motion in a *general* position, allow *a* for acceleration, correct no. of terms, condone sign errors
dM1	Use Hooke's Law to sub for the tension with extension measured from the equilibrium position and allow *a* for acceleration
A1	Correct unsimplified equation, allow *a* for acceleration
A1	Correct SHM equation and conclusion. Must use $\ddot{x}$ for acceleration **and** conclude SHM.
M1	Use of $\frac{2\pi}{\omega}$ where ω has come from an attempt at using N2L at a general point.
A1*	Obtain the given answer for the period. Must follow from fully correct working, including N2L. At least one line of working must be seen between $\ddot{x} = -\omega^2 x$ and reaching the given answer. Eg • period = $\frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{3g}{2l}}} = 2\pi\sqrt{\frac{2l}{3g}}$ • $\omega = \sqrt{\frac{3g}{2l}}$, period = $\frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2l}{3g}}$
(c)	
M1	Complete method to find the required time. Do not ISW. For example, If the sine approach is used, it must include $\frac{1}{4}T$ + their *t* value for M1. If the cos approach is used with $+\frac{2l}{3}$, it must include $\frac{1}{2}T$ – their *t* value for M1. The correct ω must be used. For the method, condone any multiple of *l* for the amplitude.
A1	Equation with at most one error
A1	Correct equation
A1	Cao